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Multi giant graviton systems, SUSY breaking and CFT

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Abstract: In this article, we describe giant gravitons in $\mathrm{AdS}_5 \times \mathcal{S}^5$ moving along generic trajectories in AdS_5. The giant graviton dynamics is solved by proving that the D3-brane effective action reduces to that of a massive point particle in AdS_5 and therefore the solutions are in one to one correspondence with timelike geodesics of AdS_5. All these configurations are related via isometries of the background, which induce target space symmetries in the world volume theory of the D-brane. Hence, all these configurations preserve the same amount of supersymmetry as the original giant graviton, i.e. half of the maximal supersymmetry. Multiparticle configurations of two or more giant gravitons are also considered. In particular, a binary system preserving one quarter of the supersymmetries is found, providing a non trivial time-dependent supersymmetric solution. A short study on the dual CFT description of all the above states is given, including a derivation of the exact induced isometry map in the CFT side of the correspondence.

Keywords: M-Theory, D-branes, AdS-CFT and dS-CFT Correspondence, Space-Time Symmetries.

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1. Introduction

The AdS/CFT duality is one of the most celebrated subjects within string theory (see [1, 2, 3] for reviews). After more than six years of continuous studies, this holographic conjecture has become more and more robust, passing all the so-called checks or theoretical experiments that the string theory community has been able to engineer. Nevertheless, there are many aspects to understand and uncover in this puzzling correspondence that relates string theory on anti-de Sitter (AdS) spaces and conformal field theories (CFT).

Recently, the study of a new kind of stable D3-brane configurations on $\mathrm{AdS}_5 \times \mathcal{S}^5$ has brought some attention. These extended solitons are stabilized by a dynamical mechanism, developing local forces on the brane that cancel its tension, avoiding the world volume collapse. More precisely, these D-brane configurations, or *giant gravitons*, correspond to D3-branes travelling on a $\mathcal{S}^1$ direction wrapping a perpendicular $\mathcal{S}^3$, both contained in the $\mathcal{S}^5$ factor of the metric, while they sit on the center of AdS_5 [4]. The dynamics of the D-brane effective action allows for two different stable solutions, one in which the radius of the $\mathcal{S}^3$ is zero and the other with a non-zero radius, bounded from above and proportional to the momentum along the $\mathcal{S}^1$ direction. These states preserve half of the supersymmetries [5] and, from the ten-dimensional point of view, their geometrical center travels along a null geodesic. Such solitons are interpreted as supergraviton states that expand into a sphere following a sort of Myers' effect [6].

Originally, these configurations where thought as the gravitational manifestation of the stringy exclusion principle [7], where the upper bound on the giant graviton momentum on the $\mathcal{S}^1$ (due to the fact that it is proportional to the radius of the $\mathcal{S}^3$ and therefore has a maximum on $\mathcal{S}^5$), is dual to the upper bound found on the conformal weight of a family of chiral operators (here the bound is easily understood from the finite rank of the gauge symmetry group) [4, 10].

Nevertheless, giant gravitons have brought new physics, like in the study of their back reaction, where particular condensates of giant gravitons result in supersymmetric solutions of type-IIB supergravity called *superstars* [9]. Another fascinating characteristic of these giant gravitons is their ability to regulate potential divergences by enlarging their size while the energy of the configuration is increased. This behavior, somehow characteristic in string theory, relates UV and IR regimes and is certainly telling us that there is a lot to understand on the reparameterization invariance in the presence of Ramond-Ramond fluxes. Also, there exists another type of solution, known as *dual giant graviton*, with the same quantum numbers, but whose world volume grows entirely in AdS_5. In this case, the dual giant graviton size does not have any upper bound and hence has been related to a different type of chiral operators in a different representation of the R-symmetry group [8].

In this article, we shall consider the generalization of giant gravitons to more complicated configurations in the AdS_5 factor of the ten-dimensional space-time. In section 2, we look for the most general solution of the D3-brane embedded in AdS_5 by making explicit the relation between the dynamics of this type of brane ansatz and point particles on AdS_5. Then, due to the fact that all the point particles move on geodesics and that all such geodesics are interrelated by isometry transformations, we obtain all the possible solutions corresponding to these *generalized giant gravitons*. In particular, we find supersymmetric configurations corresponding to giant gravitons rotating on a circular orbit in AdS_5 at a given fixed radius that depends on the angular momenta in AdS_5 and in $\mathcal{S}^5$. In section 3, we study the supersymmetry properties of the above solutions and find that all of them correspond to one half supersymmetric configurations. A family of quarter BPS solutions is proposed by considering multiparticle states of two giant gravitons. In section 4, we comment on the CFT duals and its supersymmetry properties, to end in section 5, with a short summary and conclusions.

Although the whole article is written for giant gravitons corresponding to D3-brane configurations, it is trivial to extend the supergravity discussion and calculations to the M2-brane and M5-brane giant gravitons of M-theory. Here we have not done so to avoid unnecessary complications with the notation.

Note added in proof: while we were writing this article, the work [11] appeared, with some partial overlap with the material here discussed.

2. Generalized giant gravitons and their relation with point particles in AdS_5

In this section we study the generalized giant graviton as an embedded probe D3-brane in the near horizon geometry of N D3-branes of type-IIB supergravity, i.e. on $\mathrm{AdS}_5 \times \mathcal{S}^5$, in such a way that all its spacelike directions coincide with $\mathcal{S}^5$ directions, but leaving the AdS_5 motion otherwise free. We compute the reduced action describing its classical dynamics in AdS_5 with frozen internal degrees of freedom, which turns out to be a massive point particle action. Therefore, since we are describing point particle dynamics, all possible solutions correspond to timelike geodesics in AdS_5. Note that all timelike geodesics are interrelated by isometries of the background and that these transformations correspond to symmetries of the D-brane action. After proving the above statements, we consider the particular example of a giant graviton with angular momentum on AdS_5.

2.1 Giant gravitons as point particles

First, to fix notation and conventions, we choose global coordinates on $\mathrm{AdS}_5 \times \mathcal{S}^5$ such that the AdS_5 factor of the metric is given by

$$ds^2_{\mathrm{AdS}} = -V(r)\,dt^2 + \frac{dr^2}{V(r)} + r^2\,d\Omega_3^2 \tag{2.1}$$

where the lapse function is given by $V(r) = 1 + r^2/L^2$, and the transverse three sphere can be parameterized by coordinates $(\alpha_1, \alpha_2, \psi)$, with metric

$$d\Omega_3^2 = d\alpha_1^2 + \sin^2\alpha_1 \left(d\alpha_2^2 + \sin^2\alpha_2\,d\psi^2\right). \tag{2.2}$$

Another useful set of coordinates for the three sphere is (β, ψ_1, ψ_2), with metric

$$d\Omega_3^2 = d\beta^2 + \sin^2\beta\,d\psi_1^2 + \cos^2\beta\,d\psi_2^2 \tag{2.3}$$

where now the $\mathrm{SU}(2) \times \mathrm{SU}(2)$ subgroup of the $\mathrm{SO}(4)$ rotation group is manifest. For the $\mathcal{S}^5$ part of the metric, we take

$$ds_5^2 = L^2\left(d\theta^2 + \cos^2\theta\,d\phi^2 + \sin^2\theta\,d\omega_3^2\right) \tag{2.4}$$

where

$$d\omega_3^2 = d\chi_1^2 + \sin^2\chi_1\left(d\chi_2^2 + \sin^2\chi_2\,d\chi_3^2\right) \tag{2.5}$$

is the three sphere on which the D3-brane will grow. Also, we shall use curved indices $M, N, \ldots$ for the full ten-dimensional metric and indices $\mu, \nu, \ldots$ for the AdS_5 part of the metric. A point in $\mathrm{AdS}_5 \times \mathcal{S}^5$ is then described by $X^M = (t, r, \alpha_1, \alpha_2, \psi, \theta, \phi, \chi_1, \chi_2, \chi_3)$, while its projection on AdS_5 is denoted $x^\mu = (t, r, \alpha_1, \alpha_2, \psi)$.

The D3-brane low-energy dynamics is described by a Born-Infeld action with a Chern-Simons coupling, given by

$$S_{\mathrm{D3}} = -T_3 \int d^4\sigma\,\sqrt{-g} + T_3 \int a^{[4]}, \tag{2.6}$$

where g is the pull back of the space-time metric to the world volume, i.e.

$$g_{IJ} = \partial_I X^M \partial_J X^N G_{MN}\,, \tag{2.7}$$

$a^{[4]}$ denotes the pull back of the Ramond-Ramond 4-form potential $A^{[4]}$, and we have used indices $I, J, \ldots$ for the world volume directions of the D3-brane. The tension of the brane is $T_3 = (8\pi^3 g_s \ell_s^4)^{-1}$ where g_s and ℓ_s are the coupling constant and length of the string respectively.

We want to study stable probe D-brane configurations where the D-brane has expanded on the $\mathcal{S}^5$ to a three sphere at fixed θ, while it orbits in the ϕ direction, and all the world volume modes of the D-brane are frozen. It is convenient to choose an embedding such that the world volume coordinates σ^I are identified with the appropriate space-time coordinates,

$$\begin{aligned} \sigma_1 &= \chi_1\,, & \sigma_2 &= \chi_2\,, & \sigma_3 &= \chi_3\,, \\ x^\mu &= x^\mu(\sigma_0)\,, & \theta &= \theta_0\,, & \phi &= \phi(\sigma_0)\,. \end{aligned} \tag{2.8}$$

Then, the pull back of the metric reads

$$g_{IJ} = \begin{pmatrix} G_{MN}\dot{X}^M\dot{X}^N & 0 \\ 0 & L^2\sin^2\theta (g_\chi)_{ab} \end{pmatrix}, \tag{2.9}$$

where $(g_\chi)_{ab}$ is the $\mathcal{S}^3$ metric corresponding to $d\omega_3^2$ and the dot stands for the derivative with respect to σ_0. Integrating out the spacelike directions σ_a in the D-brane action in the above embedding, we obtain the following reduced action,

$$S_{\mathrm{D3}} = -\frac{N}{L}\sin^3\theta \int d\sigma_0 \sqrt{-G_{MN}\dot{X}^M\dot{X}^N} + N\sin^4\theta_0 \int \dot{\phi}\, d\sigma_0\,. \tag{2.10}$$

Here we have used the relation $L^4 = 4\pi g_s N \ell_s^4$ characteristic of D3-brane near horizon backgrounds. A more convenient and nevertheless equivalent form for this action is

$$S = \frac{1}{2}\int d\sigma_0 \left(\frac{1}{e}G_{MN}\dot{X}^M\dot{X}^N - m^2 e\right) + N\sin^4\theta_0 \int \dot{\phi}\, d\sigma_0\,, \tag{2.11}$$

where we have defined

$$m = \frac{N}{L}\sin^3\theta\,, \tag{2.12}$$

and the *einbein* e plays the role of a Lagrange multiplier.[1] Separating the AdS$_5$ component of the motion

$$G_{MN}\dot{X}^M\dot{X}^N = g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu + L^2\cos^2\theta\,\dot{\phi}^2\,, \tag{2.14}$$

the action reads

$$S = \frac{1}{2}\int d\sigma_0 \left(\frac{1}{e}g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu - m^2 e + \frac{L^2}{e}\cos^2\theta_0\,\dot{\phi}^2 + 2N\sin^4\theta_0\,\dot{\phi}\right). \tag{2.15}$$

[1] To check the equivalence between the two actions, one can simply use the variation of S to find e,

$$e = \frac{1}{m}\sqrt{-G_{MN}\dot{X}^M\dot{X}^N} \tag{2.13}$$

and then substitute its value in S to recover S_{D3}.

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We see that the coordinate ϕ is cyclic, hence its conjugate momentum

$$p_\phi = \frac{L^2}{e}\dot{\phi}\cos^2\theta + N\sin^4\theta \tag{2.16}$$

is conserved. It is therefore useful to define the Routh function

$$\mathcal{R}(x^\mu, \dot{x}^\mu, p_\phi, \theta_0) = \dot{\phi} p_\phi - \mathcal{L}\,. \tag{2.17}$$

After some algebra, we obtain

$$\mathcal{R}(x^\mu, \dot{x}^\mu, p_\phi, \theta_0) = -\frac{1}{2e} g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu + \frac{e}{2L^2}\left[p_\phi^2 + \tan^2\theta_0\left(p_\phi - N\sin^2\theta_0\right)^2\right]. \tag{2.18}$$

Since the time derivative of θ does not appear in the routhian, the equations of motion $\partial\mathcal{R}/\partial\theta = 0$ for this coordinate yield the constraint

$$\tan\theta\left(p_\phi - N\sin^2\theta\right)\left(p_\phi - N\sin^2\theta\left(1 + 2\cos^2\theta\right)\right) = 0\,. \tag{2.19}$$

There are two stable minima for the above potential, $\theta_0 = 0$ corresponding to a collapsed D3-brane and

$$\sin^2\theta_0 = \frac{p_\phi}{N}\,, \tag{2.20}$$

representing an expanded D-brane, or giant graviton, of radius $\sin\theta_0$. Finally, there is an unstable maximum of the potential between these two. In what follows, we will restrict to the giant graviton case. We can then substitute (2.20) in the Routh function, and obtain

$$\mathcal{R} = -\frac{1}{2}\left(\frac{1}{e} g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu - e\frac{p_\phi^2}{L^2}\right). \tag{2.21}$$

Now, this is just (minus) the lagrangian for a particle of mass

$$M = \frac{p_\phi}{L} \tag{2.22}$$

moving in AdS_5.

In other words, the general solution $x^\mu(\sigma_0)$ of the dynamical problem of the above D-brane is described by the dynamics of a point particle with mass M given in eq. (2.22), which is solved by timelike geodesics.[2] Therefore, *from the AdS_5 point of view, giant gravitons are just massive point-like particles propagating along timelike geodesics.*

2.2 Generalized giant graviton solutions

To obtain the actual form of the generalized giant graviton solutions, we use the static gauge setting $\dot{x}^0 = 1$. The conjugate momenta to the remaining dynamical variables are

$$p_i = \frac{1}{e} g_{ij}\dot{x}^j\,, \tag{2.23}$$

[2] In [12], the conditions under which probe brane dynamics reduces to a particle-like one were analysed.

where $i, j, \ldots$ are the spatial directions on AdS_5, and the hamiltonian reads

$$\mathcal{H} = \dot{x}^i p_i + \mathcal{R} = \frac{V(r)}{2e} + \frac{e}{2}\left(g^{ij} p_i p_j + M^2\right). \tag{2.24}$$

The equation for e can be solved, yielding

$$e = \sqrt{\frac{V(r)}{g^{ij} p_i p_j + M^2}}\,; \tag{2.25}$$

plugging this back in the hamiltonian we obtain

$$\mathcal{H} = \sqrt{V(r)}\sqrt{g^{ij} p_i p_j + M^2}\,. \tag{2.26}$$

To make manifest the full symmetry of the problem, we use the coordinates (β, ψ_1, ψ_2) with metric (2.3) for $\mathcal{S}^3$. The explicit form of the hamiltonian is then

$$\mathcal{H} = \sqrt{V(r)}\sqrt{V(r) p_r^2 + \frac{1}{r^2}\left(p_\beta^2 + \frac{p_{\psi_1}^2}{\sin^2\beta} + \frac{p_{\psi_2}^2}{\cos^2\beta}\right) + M^2}\,. \tag{2.27}$$

Note that there is rotation symmetry in ψ_1 and ψ_2 and hence p_{ψ_1} and p_{ψ_2} are conserved. One can then easily check that J, defined by

$$J^2 = p_\beta^2 + \frac{p_{\psi_1}^2}{\sin^2\beta} + \frac{p_{\psi_2}^2}{\cos^2\beta}\,, \tag{2.28}$$

is also first integral of the hamiltonian,[3] and allows to decouple completely the angular motion from the radial one. The projection on $\mathcal{S}^3$ of the motion will always be a constant point or describe a movement on a great circle of the three-sphere. Finally, the radial motion of the brane is determined by the hamiltonian

$$\mathcal{H} = \sqrt{V(r)}\sqrt{V(r) p_r^2 + \frac{J^2}{r^2} + M^2}\,. \tag{2.29}$$

This hamiltonian is a constant of motion, and its value is the energy E of the giant graviton. This is enough to solve the one-dimensional radial motion; we have

$$p_r = \frac{E\dot{r}}{V^2(r)}\,, \tag{2.30}$$

hence

$$E^2 \dot{r}^2 = V^2(r)\left[E^2 - \left(M^2 + \frac{J^2}{r^2}\right) V(r)\right] \tag{2.31}$$

which can be readily integrated to obtain the most general solution (see for example [14] for the explicit solutions).

[3] This further conserved quantity does not descend from a spacetime isometry, but rather from a Stäckel-Killing tensor, coinciding with the Casimir invariant of any of the SU(2) subgroups of SO(4). As a consequence, $(J, p_{\psi_1}, p_{\psi_2})$ completely determine the angular motion. See [13] for further details.

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As an example, which will prove interesting in the subsequent sections, let us consider solutions with constant radius r_0. Then $p_r = 0$, and the equation of motion $\partial\mathcal{H}/\partial r = 0$ yields

$$r_0 = L\sqrt{\frac{J}{p_\phi}}\,. \tag{2.32}$$

The giant graviton rotates at constant velocity on a great circle of radius r_0 of the transverse $\mathcal{S}^3$ in AdS$_5$, with angular momentum J with projections p_{ψ_1} and p_{ψ_2} on the ψ_1 and ψ_2 axes. The total energy of this configuration is then obtained by substituting r by its value in the hamiltonian $\mathcal{H}$, to obtain

$$E = \frac{J + p_\phi}{L}\,. \tag{2.33}$$

The energy is linear in the conserved quantities, and this is reminiscent of a BPS bound. In fact, as we will show in the next section, all these solutions preserve one half of the supersymmetries.

It is interesting to see the dynamics of the D3-brane from the ten-dimensional point of view. The total hamiltonian obtained by Legendre-transforming all the canonical variables directly in the initial action (2.11) reads

$$\mathcal{H} = \frac{e}{2}\left[g^{\mu\nu}p_\mu p_\nu + \frac{1}{L^2\cos^2\theta}\left(p_\phi - N\sin^4\theta\right)^2 + \frac{N^2}{L^2}\sin^6\theta\right]. \tag{2.34}$$

This hamiltonian is linear in the Lagrange multiplier e; it is a pure constraint, imposing the vanishing of the hamiltonian. After substituting for the momenta, using equations (2.16) and (2.23), it translates into

$$G_{MN}\dot{X}^M\dot{X}^N = -\frac{e^2N^2}{L^2}\sin^6\theta < 0\,. \tag{2.35}$$

This means that each world volume element of the D3-brane follows a timelike trajectory. However, the geometrical center of the brane, which is located at $\theta = 0$, follows a null trajectory in the full ten-dimensional space-time.

2.3 Anti-de Sitter isometries and geodesics

In the above subsection, we have shown that giant gravitons follow timelike geodesics in AdS$_5$. Also, it is important to note that in a homogeneous spacetime, timelike geodesics can be mapped one into another by means of isometry transformations. This can be easily proved using the following standard proposition (see for example [15])

Proposition 1. *Let $e_1,\ldots,e_5$ and $f_1,\ldots,f_5$ be tangent frames on AdS$_5$ at points p and q, respectively. Then there is a unique isometry $\phi : \mathbb{R}^4_2 \to \mathbb{R}^4_2$ carrying AdS$_5$ isometrically to itself, with $\phi(p) = q$ and $\phi_*(e_i) = f_i$ for $i = 1\ldots5$.*

Then, using the fact that any geodesic is uniquely determined by a point p and a unit timelike tangent vector, we find,

Theorem 1. *Every timelike geodesic of AdS_5 can be mapped on any other timelike geodesic by an isometry.*

As a consequence, an alternative way to construct the generic giant graviton solution of the previous subsection is to start with any given particular solution and transform it by acting with an AdS_5 isometry.

We would like to stress that the above isometries of the background translate into target space symmetries of the world volume theory of the D-brane, and therefore map solutions into solutions. In fact, we will argue in the next section that this target space symmetry leaves the full supersymmetric action invariant, and hence all these solutions preserve the same amount of supersymmetry.

To make our discussion more specific, we introduce in this subsection a mathematical formalism to handle the isometries in a pleasant form that will prove useful in forthcoming sections.

AdS_5 is a homogeneous five dimensional manifold that can be embedded as the hyperboloid

$$-X_0^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2 - X_5^2 = -L^2 \tag{2.36}$$

in flat six-dimensional space-time with signature $+2$,

$$\mathrm{d}s^2 = -\mathrm{d}X_0^2 + \mathrm{d}X_1^2 + \mathrm{d}X_2^2 + \mathrm{d}X_3^2 + \mathrm{d}X_4^2 - \mathrm{d}X_5^2\,. \tag{2.37}$$

The hyperboloid is manifestly invariant under the $\mathrm{SO}(4,2)$ isometry group of the embedding space, and can be parameterized with coordinates $(\tau, \rho, \alpha_1, \alpha_2, \psi)$ such that

$$\begin{aligned}
X_0 &= L\cosh\rho\cos\tau\,, & X_5 &= L\cosh\rho\sin\tau\,,\\
X_1 &= L\sinh\rho\cos\alpha_1\,, & X_2 &= L\sinh\rho\sin\alpha_1\cos\alpha_2\,,\\
X_3 &= L\sinh\rho\sin\alpha_1\sin\alpha_2\cos\psi\,, & X_4 &= L\sinh\rho\sin\alpha_1\sin\alpha_2\sin\psi\,.
\end{aligned} \tag{2.38}$$

In these coordinates, the induced AdS_5 metric reads[4]

$$\mathrm{d}s^2 = L^2\left(-\cosh^2\rho\;\mathrm{d}\tau^2 + \mathrm{d}\rho^2 + \sinh^2\rho\;\mathrm{d}\Omega_3^2\right) \tag{2.39}$$

where $\mathrm{d}\Omega_3^2$ is the three-sphere metric given in equation (2.2). Here $\tau \in [0, 2\pi)$, $\rho \in \mathbb{R}_+$, $\alpha_i \in [0, \pi)$, $\psi \in [0, 2\pi)$. To avoid closed timelike curves, we take the universal cover of the hyperboloid, by simply allowing the time coordinate τ to range on the whole real axis.

The following propositions [15] give a geometrical picture of the geodesics on this hyperboloid,

Proposition 2. *Let γ be a nonconstant geodesic of $AdS_5 \subset \mathbb{R}_2^4$. If γ is spacelike it is a parameterization of one branch of a hyperbola in $\mathbb{R}_2^4$. If γ is null, it is a straight line, that is, a geodesic of $\mathbb{R}_2^4$. If γ is timelike it is a periodic parameterization of an ellipse in $\mathbb{R}_2^4$.*

[4] In this subsection, we use here the new dimensionless coordinates ρ and τ, defined as $L\,\mathrm{d}\rho = V^{-1}(r)\,\mathrm{d}r$ and $\tau = t/L$.

Proposition 3. *The geodesics of* $AdS_5 \subset \mathbb{R}^4_2$ *are the curves sliced from* AdS_5 *by planes* Π *through the origin of* $\mathbb{R}^4_2$.

To give an explicit example of solution construction by isometries, we shall work out the case of the giant graviton moving at constant radius on the equatorial plane $\alpha_1 = \pi/2$, $\alpha_2 = \pi/2$ found in the previous subsection. From the embedding space point of view, we restrict to the subspace $\{X_1 = 0, X_2 = 0\}$. The points in the equatorial plane form the AdS_3 hyperboloid

$$- X_0^2 + X_3^2 + X_4^2 - X_4^2 = -L^2 \,. \tag{2.40}$$

Hence, a point $x \in \mathrm{AdS}_3$ can be parameterized by the $\mathrm{SL}(2,\mathbb{R})$ matrix [16]

$$\mathbb{X} = \frac{1}{L} \begin{pmatrix} X_0 + X_3 & X_5 + X_4 \\ -X_5 + X_4 & X_0 - X_3 \end{pmatrix}, \tag{2.41}$$

because $\det \mathbb{X} = 1$ is equivalent to condition (2.40). In terms of the coordinates, the explicit matrix is

$$\mathbb{X}(\tau,\rho,\psi) = \begin{pmatrix} \cosh\rho\cos\tau + \sinh\rho\cos\psi & \cosh\rho\sin\tau + \sinh\rho\sin\psi \\ -\cosh\rho\sin\tau + \sinh\rho\sin\psi & \cosh\rho\cos\tau - \sinh\rho\cos\psi \end{pmatrix}. \tag{2.42}$$

The residual isometry group is $\mathrm{SO}(2,2) \subset \mathrm{SO}(4,2)$. Using the isomorphism $\mathrm{SO}(2,2) \cong \mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R})/\mathbb{Z}_2$, its action on the point $\mathbb{X}$ on the equatorial plane is given by

$$(\rho_L, \rho_R) : \mathbb{X} \mapsto \rho_L \mathbb{X} \rho_R \tag{2.43}$$

with $\rho_L, \rho_R \in \mathrm{SL}(2,\mathbb{R})$, and the $\mathbb{Z}_2$ quotient is obtained by the identification of (ρ_L, ρ_R) with $(-\rho_L, -\rho_R)$. A basis of the $sl(2,\mathbb{R})$ algebra is given by the matrices

$$\begin{gathered} \mathbf{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \gamma_0 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad \gamma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \gamma_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\ \gamma_a\gamma_b = \eta_{ab}\mathbf{1} - \varepsilon_{ab}{}^c\gamma_c \,, \qquad \eta_{ab} = \mathrm{diag}(-1,1,1)\,, \qquad \varepsilon^{012} = 1\,, \end{gathered} \tag{2.44}$$

which generate the following elements of the group,

$$\mathcal{R}(\alpha) = e^{\alpha\gamma_0}\,, \qquad \mathcal{S}(\alpha) = e^{\alpha\gamma_1}\,, \qquad \mathcal{T}(\alpha) = e^{\alpha\gamma_2}\,. \tag{2.45}$$

A useful relation is $\mathcal{S}(\alpha) = \mathcal{R}(-\pi/4)\,\mathcal{T}(\alpha)\mathcal{R}(\pi/4)$. Now (τ,ρ,ψ) are essentially the Euler angles of $\mathrm{SL}(2,\mathbb{R})$,

$$\mathbb{X} = e^{\frac{1}{2}(\tau-\psi)\gamma_0} e^{\rho\gamma_1} e^{\frac{1}{2}(\tau+\psi)\gamma_0} \tag{2.46}$$

and any point of the equatorial plane may be written

$$\mathbb{X}(\tau,\rho,\sigma) = \mathcal{R}\left(\frac{\tau-\psi}{2}\right)\mathcal{T}(\rho)\mathcal{R}\left(\frac{\tau+\psi}{2}\right). \tag{2.47}$$

Let us start with the original giant graviton sitting in the center of AdS_5 found in ref. [4]. This giant graviton is described by the timelike geodesic $\rho = 0$ (a particle at rest in the center of AdS_5). We can take for convenience $\psi = 0$, then its trajectory is parameterized by

$$\mathbb{X}(\lambda) = \mathcal{R}(\lambda)\,, \qquad \lambda \in \mathbb{R}\,. \tag{2.48}$$

Next consider the isometry $(\mathcal{T}(\rho_0), \mathbf{1})$. This transformation maps the old geodesic into the new geodesic

$$\mathbb{X}'(\lambda) = \mathcal{T}(\rho_0)\mathcal{R}(\lambda)\,. \tag{2.49}$$

From eq. (2.47), we can rewrite the trajectory as $\mathbb{X}'(\lambda) = \mathbb{X}(\lambda, \rho_0, \lambda)$ and hence $\{\psi = \tau, \rho = \rho_0\}$, i.e. the particle moves on a circle of radius $\rho = \rho_0$ at constant velocity $\partial_t \psi = 1/L$. In this particular case, from the hamiltonian analysis, we get that $J = p_\psi = Er^2/LV(r)$. Then, using equation (2.33), we find the expected value for the radius, in agreement with (2.32).

From the point of view of the embedding space, the isometry transformation $(\mathcal{T}(\rho_0), \mathbf{1})$ is the composition of two hyperbolic rotations in the $(0,3)$ and $(4,5)$ planes, both with angle ρ_0:

$$(\mathcal{T}(\rho_0), \mathbf{1}) = R_{(0,3)}(\rho_0) \circ R_{(4,5)}(\rho_0)\,. \tag{2.50}$$

Note that the two planes are orthogonal and the rotations commute. Moreover, the X_1 and X_2 coordinates are left untouched by this transformation, hence

$$X_1' = X_1\,, \qquad X_2' = X_2\,, \tag{2.51}$$

and the corresponding infinitesimal generator is given by

$$T = X_0\partial_3 + X_3\partial_0 + X_4\partial_5 + X_5\partial_4\,. \tag{2.52}$$

In particular, the above infinitesimal transformation will be of relevance in the study of the dual operators in the CFT.

3. One half and less supersymmetric states

In this section, we study the supersymmetry properties of generalized giant graviton configurations in $\mathrm{AdS}_5 \times \mathcal{S}^5$. Then, we consider particular examples starting with single particle states, to end with multiparticle states, having in mind the idea of breaking some additional supersymmetry.

Bosonic D-brane and M-brane configurations living in bosonic supersymmetric backgrounds are supersymmetric if the background Killing spinor ϵ satisfies the κ-symmetry constraint

$$(1 + \Gamma)\,\epsilon = 0\,, \tag{3.1}$$

where Γ is the relevant κ-symmetry matrix [17, 18, 19]. The above equation dictates the form and number of real independent parameters that produce supersymmetric transformations on the world volume theory of the brane.

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The background Killing spinor can always be written as $\Pi(x)\epsilon_0$, with $\Pi(x)$ a general space-time dependent matrix. Its rank is constant and equal to the number of supersymmetry generators which leave invariant the spacetime fields; the rank of the matrix $M(x) \equiv (1-\Gamma(x))\Pi(x)$ counts the number of surviving world volume supersymmetries.

Note that isometry transformations of the background translate into target space symmetries of the world volume theory, therefore mapping solutions into physically distinct solutions. The κ-symmetry condition (3.1) is covariant under general diffeomorphisms, in particular it is left invariant in form under isometries. Also, $M(x)$ transforms as a scalar field under general diffeomorphisms and if we act with an isometry its rank remains unchanged. Therefore, *the transformed brane solutions preserve the same amount of supersymmetries.*

The general giant graviton solutions found in the previous section are related via isometries to the giant graviton solution sitting in the center of AdS_5, which is known to be one half supersymmetric [5]. Consequently, applying the previous argument, we deduce that *all generalized giant gravitons are half BPS states.*

Nevertheless, the particular form of the surviving supersymmetry generator depends on the specific form of the solution. Accordingly, to obtain configurations with less preserved supersymmetry, we can consider multiparticle giant graviton states, such that the relative motion breaks some further fraction of supersymmetry. Observe that this strategy to construct fractional BPS states using isometries, is not peculiar to giant gravitons but can be applied to any multibrane configuration.

In what follows, we will illustrate the above arguments with specific cases in order to provide explicit examples.

3.1 Giant graviton with angular momentum on AdS_5

Let us consider the solution describing a giant graviton located at constant radial position r_0 given in eq. (2.32) and rotating on one of the great circles of the $\mathcal{S}^3$, with constant angular velocity $\dot{\psi} = 1/L$. The corresponding embedding is

$$\begin{aligned} &\sigma_0 \equiv \tau = t\,, \qquad \sigma_1 = \chi_1\,, \qquad \sigma_2 = \chi_2\,, \qquad \sigma_3 = \chi_3\,,\\ &r = r_0\,, \qquad \psi = \frac{\tau}{L}\,, \qquad \theta = \theta_0\,, \qquad \phi = \frac{\tau}{L}\,,\\ &\alpha_1 = \alpha_2 = \frac{\pi}{2}\,. \end{aligned} \tag{3.2}$$

where the values of (r,θ) can be parameterized by the corresponding angular momenta on AdS_5 and $\mathcal{S}^5$ as

$$p_\phi = N\sin^2\theta_0\,, \qquad \frac{J}{p_\phi} = \left(\frac{r_0}{L}\right)^2. \tag{3.3}$$

To write the κ-symmetry constraint we need to set the following definitions and conventions: we label ten-dimensional tangent space indices by $A, B, \ldots$, such that the vielbein is written as e_M^A; a particular value of a Lorentz index is underlined, e.g. $\underline{\psi}$, while curved space-time indices are left unadorned. We use a real representation of the ten-dimensional Γ-matrices Γ_A such that $\{\Gamma_A, \Gamma_B\} = 2\eta_{AB}$, where $\eta_{AB} = \mathrm{diag}(-1, 1, \ldots, 1)$ and $\Gamma_{A_1A_2\ldots A_n}$

is a completely antisymmetrized on indices $(A_1, A_2, \ldots, A_n)$ with weight one. Finally, we combine the two real Majorana-Weyl Killing spinors of type-IIB supergravity into a single complex Majorana-Weyl spinor ϵ, that satisfies the following Killing condition

$$\left(D_M - \frac{i}{4}(\gamma_5 + \gamma)\Gamma_M\right)\epsilon = 0\,, \tag{3.4}$$

where D_M is the ten-dimensional covariant derivative, $\gamma_5 = \Gamma^{\underline{\theta\phi\chi_1\chi_2\chi_3}}$ and $\gamma = \Gamma^{\underline{t\rho\alpha_1\alpha_2\psi}}$. The solution to this equation is[5]

$$\begin{aligned}\epsilon = &\left[e^{\frac{i}{2}\theta\gamma_5\Gamma^{\underline{\theta}}} e^{\frac{i}{2}\phi\gamma_5\Gamma^{\underline{\phi}}} e^{-\frac{i}{2}\chi_1\Gamma^{\underline{\chi_1\theta}}} e^{-\frac{i}{2}\chi_2\Gamma^{\underline{\chi_2\chi_1}}} e^{-\frac{i}{2}\chi_3\Gamma^{\underline{\chi_3\chi_2}}} \times\right.\\ &\left.\times\, e^{\frac{i}{2}\alpha\Gamma^{\underline{r}}\gamma} e^{-\frac{it}{2L}\Gamma^{\underline{t}}\gamma} e^{-\frac{1}{2}\alpha_1\Gamma^{\underline{\alpha_1 r}}} e^{-\frac{1}{2}\alpha_2\Gamma^{\underline{\alpha_2\alpha_1}}} e^{-\frac{1}{2}\psi\Gamma^{\underline{\psi\alpha_2}}}\right]\epsilon_0\,, \end{aligned} \tag{3.5}$$

where $\sinh\alpha = r/L$ and ϵ_0 is a general constant complex spinor.

With the above definitions, in the particular case of a D3-brane, the κ-symmetry constraint (3.1) becomes

$$(1 - i\Gamma)\,\epsilon = 0\,, \tag{3.6}$$

where

$$\Gamma = \frac{1}{4!}\varepsilon^{IJKL}\Gamma_{IJKL}\,, \qquad \Gamma_I = \partial_I X^M e_M^A \Gamma_A\,. \tag{3.7}$$

Using the embedding (3.2), we get the equation

$$\left[\sqrt{1 + \left(\frac{r}{L}\right)^2}\,\Gamma_{\underline{t\chi_1\chi_2\chi_3}} + \left(\frac{r}{L}\right)\Gamma_{\underline{\psi\chi_2\chi_3}} + \cos\theta\Gamma_{\underline{\phi\chi_2\chi_3}} - i\sin\theta\right]\epsilon = 0\,. \tag{3.8}$$

This expression can be simplified using the following relations

$$\begin{aligned} \Gamma_{\underline{t\chi_1\chi_2\chi_3}} &= -\gamma^5\Gamma^{\underline{\theta}}\Gamma_{\underline{t\phi}}\,, \qquad \Gamma_{\underline{\psi\chi_2\chi_3}} = -\gamma^5\Gamma^{\underline{\phi}}\Gamma_{\underline{\psi\phi}}\,, \qquad \Gamma_{\underline{\phi\chi_2\chi_3}} = \gamma^5\Gamma^{\underline{\theta}}\,,\\ \cos\theta - i\sin\theta\gamma\Gamma^{\underline{\phi}} &= e^{-i\theta\gamma\Gamma^{\underline{\phi}}}\,, \qquad \cos\alpha\Gamma_{\underline{t\phi}} - \sin\alpha\Gamma_{\underline{\psi\phi}} = \Gamma_{\underline{t\phi}}e^{-\alpha\Gamma_{\underline{t\psi}}}\,, \end{aligned} \tag{3.9}$$

to obtain

$$\left[\Gamma^{\underline{t\phi}}e^{\alpha\Gamma^{\underline{t\psi}}} + e^{-i\theta\gamma^5\Gamma^{\underline{\theta}}}\right]\epsilon = 0\,. \tag{3.10}$$

Next, we pull the above operator through the space-time dependent part of ϵ (see eq. 3.5), using the Γ-matrix algebra. After a long but straightforward calculation, we arrive to

$$\left[\left(\sinh^2\alpha + \sinh\alpha\cosh\alpha e^{-(\frac{it}{L}\Gamma^{\underline{t}}\gamma - \psi\Gamma^{\underline{\psi\alpha_2}})}\Gamma^{\underline{t\psi}}\right)(1 - i\Gamma^{\underline{r\alpha_1}}) + \left(1 + \Gamma^{\underline{t\phi}}\right)\right]\epsilon_0 = 0\,. \tag{3.11}$$

At first sight, it may appear that only one quarter of the supersymmetries will survive, demanding that both projectors

$$P_{t\phi} = \frac{1}{2}(1 + \Gamma^{\underline{t\phi}}) \qquad \text{and} \qquad P_{r\alpha_1} = \frac{1}{2}(1 - i\Gamma^{\underline{r\alpha_1}}) \tag{3.12}$$

[5]See for example [5]. Due to conventions, some signs are different.

annihilate the spinor ϵ_0. However, the general solution to this equation is found by decomposing the spinor ϵ_0 into four independent components defined by the above projectors, i.e.

$$\epsilon_0 = \epsilon^{++} + \epsilon^{+-} + \epsilon^{-+} + \epsilon^{--}\,, \tag{3.13}$$

where

$$P_{t\phi}\epsilon^{+\pm} = \epsilon^{+\pm}\,, \qquad P_{t\phi}\epsilon^{-\pm} = 0\,, \qquad P_{r\alpha_1}\epsilon^{\pm+} = \epsilon^{\pm+}\,, \qquad P_{r\alpha_1}\epsilon^{\pm-} = 0\,. \tag{3.14}$$

Using the relation $\psi = t/L$ and more Γ-matrix algebra, we get

$$\cosh\alpha\left(\cosh\alpha + \sinh\alpha\Gamma^{\underline{t\psi}}\right)\epsilon^{+-} + \sinh\alpha\left(\sinh\alpha + \cosh\alpha\Gamma^{\underline{t\psi}}\right)\epsilon^{--} + \epsilon^{++} = 0\,. \tag{3.15}$$

Projecting this equation with $P_{\rho\alpha_1}$, we obtain $\epsilon^{++} = 0$, and hence the supersymmetry condition reduces to

$$\cosh\alpha\left(\cosh\alpha + \sinh\alpha\Gamma^{\underline{t\psi}}\right)\epsilon^{+-} + \sinh\alpha\left(\sinh\alpha + \cosh\alpha\Gamma^{\underline{t\psi}}\right)\epsilon^{--} = 0\,. \tag{3.16}$$

From this equation we can read off the final conditions on the Killing spinor, i.e.

$$\epsilon^{++} = 0 \qquad \text{and} \qquad \epsilon^{+-} = \tanh\alpha\Gamma^{\underline{\psi t}}\epsilon^{--}\,. \tag{3.17}$$

Therefore, ϵ^{-+} and ϵ^{--} are unconstrained and parameterize, as expected, a total of $8+8=16$ independent supersymmetries: the solution preserves exactly half of the supersymmetries.

3.2 One quarter BPS states as binary systems

We have found that any giant graviton moving in AdS_5 behaves like a half BPS particle. Therefore, no new breaking of supersymmetry occurs by allowing angular momenta in AdS_5 and/or radial time dependence. To obtain smaller fractions of supersymmetry, what we can certainly do is to consider the case of two or more giant gravitons travelling along different geodesics on AdS_5. To make the calculation tractable, we shall require that the ten-dimension distance between the giant gravitons remains always larger than the string length, in such a way that the abelian probe-brane approximation holds. In general, we expect that no supersymmetry would survive. Nevertheless, fine tuning the configurations may lead to one quarter or smaller fractions of residual supersymmetry.

For example, let us consider a configuration of two giant gravitons, such that the first one sits at $r = 0$ while the second orbits at $r = r_0$. In this binary system, there is a net angular momentum between the two giant gravitons that cannot be removed by any isometry.

The supersymmetry of such a configuration is easily checked in the abelian regime, where we can neglect the interactions between the two probe D-branes. Then, the total hamiltonian is just the sum of each hamiltonian, and two different supersymmetry constraints have to be imposed on the Killing spinor ϵ. Following previous works [5], it is not difficult to see that the relevant projector for the first giant graviton located at $r = 0$ is

$$P_{t\phi}\epsilon_0 = 0\,. \tag{3.18}$$

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Using the decomposition (3.13) of ϵ_0 by projectors $P_{t\phi}$ and $P_{r\alpha_1}$, we get that

$$\epsilon^{+\pm} = 0\,, \tag{3.19}$$

while for the second giant graviton, we found in eq. (3.17)

$$\epsilon^{++} = 0 \qquad \text{and} \qquad \epsilon^{+-} = \tanh\alpha\,\Gamma^{\underline{\psi t}}\epsilon^{--}\,. \tag{3.20}$$

Since the two giant gravitons share the projector $P_{t\phi}$, we see immediately that to solve both conditions simultaneously we must impose $\epsilon^{--} = 0$ in addition to (3.19). Therefore only ϵ^{-+} is unconstrained, meaning that only eight real supersymmetries survive, or simply that one quarter of the original supersymmetry is preserved.

It would be interesting to proceed further and use this technique to obtain more exotic configurations, with smaller fractions of supersymmetry. For example, one could try to construct solutions preserving one eighth of the supersymmetries by adding to the above binary system a third D-brane projecting out another half of the residual supersymmetries. As we will see shortly, such states may have interesting CFT duals, but go beyond the scope of the present work and we leave them for future investigations.

4. Comments on the dual CFT picture

In this section we review some known facts about the AdS/CFT duality in order to study the generalized giant graviton from the CFT side. Then we make a few comments on the meaning of an isometry of AdS_5 from the point of view of the CFT, and how this transformation acts on chiral primary fields and on the supercharges. Once this is clarified, we describe how the candidate for the giant graviton dual operator transforms. As a last point, we take the obvious and naive candidate operator describing the multiparticle state of two giant gravitons, and argue that it preserves one quarter of the total supersymmetry, as it should be from the supergravity picture.

4.1 Giant gravitons and CFT operators

The remarkable conjecture of Maldacena, relating type-IIB superstring theory on $\mathrm{AdS}_5 \times \mathcal{S}^5$ with $\mathcal{N} = 4$ super Yang-Mills (SYM) in four dimensions has been successfully put under several tests until now. Both theories have different realizations of the same superconformal group. In $\mathrm{AdS}_5 \times \mathcal{S}^5$ we have the isometry group $\mathrm{SO}(2,4) \times \mathrm{SO}(6)$ or better its covering group $\mathrm{SU}(2,2) \times \mathrm{SU}(4)$ (since spinors are also involved on this background) and there are thirty-two real supercharges that enhance the invariance group to the supergroup $\mathrm{SU}(2,2|4)$. On the field theory side, the $\mathrm{SU}(2,2)$ part is realized as the conformal group of flat four-dimensional Minkowski space-time, while the $\mathrm{SU}(4)$ part corresponds to the R-symmetry group. Although, at first sight, there are only sixteen real supercharges Q, the extension to the superconformal group provides the necessary sixteen extra real supercharges S, to reach the grand total of thirty-two real supercharges.

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The checks of this conjecture are mostly restricted to the strong coupling limit of the 't Hooft coupling constant, in the large-N approximation of the SYM theory, corresponding to the supergravity regime of superstring theory. In this limit, the analysis of Kaluza-Klein excitations due to compactification on $\mathcal{S}^5$ leads to several families of field modes with well-defined transformation properties under the $SU(2,2|4)$ group. At this point, a study of superconformal representations is needed, since the conjecture translates into a series of predictions concerning the spectrum of SYM operators. In particular, *short* representations are specially useful due to the fact that some of their properties are protected from quantum corrections. In fact, chiral fields (fields belonging to these representations) in SYM theory correspond to Kaluza-Klein harmonics on the gravity side.

Primary fields are defined as fields annihilated by all supercharge operators S and all generators of special conformal transformations K at the origin. Chiral primary fields are additionally annihilated by some of the Q. For example, we construct the half BPS family by considering symmetric traceless combination of the scalar fields Φ^I of $\mathcal{N}=4$ SYM of the form $\mathcal{O}^{I_1 \dots I_n} = \mathrm{Tr}\left(\Phi^{(I_1} \cdots \Phi^{I_n)}\right)$. These operators have protected scaling dimension Δ, coinciding with their R-symmetry charge; Φ^I is in the **6** representation of the R-symmetry group $SU(4)$ and therefore $\mathcal{O}^{I_1 \dots I_n}$ has weight $(0,n,0)$, which matches precisely one of the unitarity bounds for short representations of the superconformal group.[6] The full chiral multiplet is generated by the repeated action of the operators Q and P on the chiral primary. All the multiplet is annihilated by some of the Q and, due to the structure of the superconformal algebra $[Q,K] \sim S$, half of the S also give zero on all the states of the multiplet, recovering in this way the notion of sixteen conserved real supersymmetries, i.e. eight generated by the Q and eight by the S.

Normally, single trace operators in the CFT side are related to single particle states in the gravity side since, in the large-N limit, single trace operators form an orthogonal set. Nevertheless, this is only correct if the R-symmetry charge of the single trace operators is not comparable with N. If this is not the case, the orthogonality property is lost, and we have to use a different type of operators to describe the corresponding dual single particle states. Giant gravitons are among this type of particles with very high R-charge. Therefore, they are not expected to be described by single trace operators. In [10], subdeterminant operators of the SYM theory were proposed as the main candidates to describe the original giant graviton sitting at the center of AdS_5,

$$\det{}_n(\Phi) = \frac{1}{n!} \varepsilon_{i_1 \dots i_{N-n} j_1 \dots j_n} \varepsilon^{i_1 \dots i_{N-n} k_1 \dots k_n} \Phi^{j_1}_{k_1} \cdots \Phi^{j_n}_{k_n} \,, \tag{4.1}$$

where, in the above expression, we have written explicitly the $SU(N)$ indices but neglected the R-symmetry ones. These operators have the correct orthogonality property when n is comparable to N and therefore are good candidates to describe single particle states. They belong to a short representation preserving half of the total supersymmetry, more precisely to a chiral family of $SU(4)$ with $(0,n,0)$ weight. Note that these operators reproduce the correct bound for the R-charge, saturated by giant gravitons with $n = N$.

[6] See [20] for a short review.

One of the possible approaches to find the form of the dual SYM operator for generalized giant gravitons consists on constructing the induced dual map on the SYM theory to an isomorphism on AdS_5 and then apply it on a subdeterminant operator to obtain the desired generalized dual operator. Once the new CFT dual operator is obtained, one can check that the resulting properties, like for example supersymmetry, are the expected ones.

4.2 Isometries and induced CFT transformations

To obtain the form of the induced dual map, we project the isometry into the boundary of AdS_5. This boundary can be obtained by considering very large values of the X_i in the embedding (2.36). Taking $X_i = R\tilde{X}_i$ in the limit $R \to \infty$, this condition becomes

$$\tilde{X}_0^2 - \tilde{X}_1^2 - \tilde{X}_2^2 - \tilde{X}_3^2 - \tilde{X}_4^2 + \tilde{X}_5^2 = 0 \tag{4.2}$$

and the boundary is given by the projective equivalence classes $X_i \sim tX_i$, $t \in \mathbb{R}$. Using this identification, we can rescale the coordinates such that $X_0^2 + X_5^5 = 1$ and hence the boundary of AdS_5 is given by

$$X_0^2 + X_5^5 = 1 = X_1^2 + X_2^2 + X_3^2 + X_4^2\,, \tag{4.3}$$

i.e. it is just $\mathcal{S}^1 \times \mathcal{S}^3$, with the lorentzian induced metric.

It is convenient to use the euclidean version of AdS_5 to formulate the AdS/CFT correspondence. To do this, we rotate $X_5 \mapsto iX_5$, mapping AdS_5 into the five-dimensional ball B_5,

$$-X_0^2 + X_5^2 + \sum_i (X_i)^2 = -L^2\,, \qquad i = 1\dots 4\,. \tag{4.4}$$

To reach the boundary, we take $X_a \to \infty$ and define $X_a = t\tilde{X}_a$. Then, the boundary is given by $-\tilde{X}_0^2 + \tilde{X}^5 + \sum_i (\tilde{X}_i)^2 = 0$ with the identification $\tilde{X} \sim \lambda\tilde{X}$. The new coordinates $\tilde{u} = \tilde{X}_0 + \tilde{X}_5$ and $\tilde{v} = \tilde{X}_0 - \tilde{X}_5$ are such that $\tilde{u}\tilde{v} = \tilde{X}_i\tilde{X}_i$, and we can use the projective equivalence to set $\tilde{v} = 1$. Then $\tilde{u} = X_iX_i$, and the boundary is spanned by the coordinates X_i, $i = 1\dots 4$, endowed with the euclidean metric.

Let us see how isometry transformations are mapped to the boundary of euclidean AdS_5 in these coordinates. We shall restrict, for sake of simplicity, to infinitesimal isometries. The finite transformations can in any case be recovered by exponentiation. Transformations generated by the hamiltonian (time translations) are generated by the infinitesimal transformation

$$X_0' = X_0 + \varepsilon X_5\,, \qquad X_5' = X_5 + \varepsilon X_0\,, \qquad X_i' = X_i\,. \tag{4.5}$$

The induced transformation on the boundary is then

$$\tilde{u}' = (1+\varepsilon)\tilde{u}\,, \qquad \tilde{v}' = 1 - \varepsilon\,, \qquad \tilde{X}_i' = \tilde{X}_i\,, \tag{4.6}$$

and we multiply by $(1+\varepsilon)$ to choose the equivalence class representative with $\tilde{v}' = 1$,

$$\tilde{u}' = (1+2\varepsilon)\tilde{u}\,, \qquad \tilde{v}' = 1\,, \qquad \tilde{X}_i' = (1+\varepsilon)\tilde{X}_i\,. \tag{4.7}$$

Hence, on the boundary, the AdS hamiltonian generates a dilatation $X_i' = \lambda X_i$, and the energy is mapped in the conformal weight of the dual operator $\mathcal{O}$.

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Consider now the transformation $R_{(0,3)}(\rho_0) \circ R_{(4,5)}(\rho_0)$ which maps the $\rho = 0$ geodesic to the geodesic $\{\psi = t, \rho = \rho_0\}$. The infinitesimal transformation reads

$$\begin{aligned} X_0' &= X_0 + \varepsilon X_3\,, & X_3' &= X_3 + \varepsilon X_0\,, \\ X_4' &= X_4 + \varepsilon X_5\,, & X_5' &= X_5 - \varepsilon X_4\,, \\ X_1' &= X_1\,, & X_2' &= X_2\,, \end{aligned} \tag{4.8}$$

and acts on the boundary as

$$\begin{aligned} u' &= u + \varepsilon(X_3 - X_4) & v' &= 1 + \varepsilon(X_3 + X_4) \\ X_3' &= X_3 + \frac{\varepsilon}{2}(u+v) & X_4' &= X_4 + \frac{\varepsilon}{2}(u-v) \\ X_1' &= X_1\,, & X_2' &= X_2\,. \end{aligned} \tag{4.9}$$

Multiplying by $1 - \varepsilon(X_3 + X_4)$ to take $v' = 1$,

$$\begin{aligned} u' &= u + \varepsilon(X_3 - X_4) - \varepsilon u(X_3 + X_4)\,, & v' &= 1 \\ X_3' &= X_3 + \frac{\varepsilon}{2}(u+v) - \varepsilon X_3(X_3 + X_4)\,, & X_1' &= (1 - \varepsilon u(X_3 + X_4))X_1\,, \\ X_4' &= X_4 + \frac{\varepsilon}{2}(u-v) - \varepsilon X_4(X_3 + X_4)\,, & X_2' &= (1 - \varepsilon u(X_3 + X_4))X_2\,, \end{aligned} \tag{4.10}$$

and using $v = 1$, $u = (X_i)^2$, we obtain the action of the transformation on the boundary,

$$\begin{aligned} X_1' &= [1 - \varepsilon(X_3 + X_4)]\, X_1\,, \\ X_2' &= [1 - \varepsilon(X_3 + X_4)]X_2\,, \\ X_3' &= [1 - \varepsilon(X_3 + X_4)]X_3 + \frac{\varepsilon}{2}(X^2 + 1)\,, \\ X_4' &= [1 - \varepsilon(X_3 + X_4)]X_4 + \frac{\varepsilon}{2}(X^2 - 1)\,. \end{aligned} \tag{4.11}$$

Remember that the general infinitesimal $\mathrm{SO}(5,1)$ transformation can be parameterized as

$$X_i' = (1 + \lambda - \beta \cdot X)X_i + \frac{X^2}{2}\beta_i + \frac{1}{2}\alpha_i + \omega_{ij}X_j\,, \tag{4.12}$$

with α_i, ω_{ij}, λ and β_i the infinitesimal parameter generating the translations, rotations, dilatations and special conformal transformations respectively. Hence, we see that the transformation under consideration is obtained with

$$\alpha_i = (0,0,\varepsilon,-\varepsilon)\,, \qquad \omega_{ij} = 0\,, \qquad \lambda = 0\,, \qquad \beta_i = (0,0,\varepsilon,\varepsilon)\,, \tag{4.13}$$

i.e. it is a combination of a translation with a special conformal transformation.

4.3 Generalized giant gravitons and transformed subdeterminants

Once we know the CFT form of the induced transformation corresponding to an isometry of AdS_5, we just have to consider its action on the dual operator (4.1) of a giant graviton sitting on the center of AdS_5 to obtain the form of the dual operator of the generalized giant graviton. Since this induced transformation is an element of the conformal group, we know how it acts on any CFT operator. For example, on field realizations that are eigenfunctions of the dilatation operator with scaling dimension Δ, like $\Phi^I(x)$, we have

$$\Phi'^I(x') = \left|\frac{\partial x'}{\partial x}\right|^{-\Delta/4} \Phi^I(x)\,, \qquad x' = \Lambda(x)\,, \tag{4.14}$$

where the prime indicates transformed quantities and Λ is the induced conformal map on the four-dimensional space-time coordinates. In particular, consider the infinitesimal transformation of equation (4.11) for the above scalar fields $\Phi^I(x)$ evaluated at $x = 0$,

$$\Phi'^I(0) = (1 - \alpha^i \partial_i)\Phi^I(0)\,. \tag{4.15}$$

Due to the fact that subdeterminants are made out of these fields, as shown in (4.1), their infinitesimal transformation is just the same. Note that the transformed operator has the same conformal weight as the original operator, in agreement with the supergravity picture where the momentum along the $\mathcal{S}^5$ directions has not been modified. The action of the map is just to add a descendent field part to the original operator, as should be the case for a bosonic transformation acting on a representation of the superconformal group. Also, note that in the transformed subdeterminant a space-time scale α that was absent before appears, signing the fact that our giant graviton is not any more in the center on AdS_5.

The supersymmetries preserved by transformed subdeterminants have to be the same as the ones of the original subdeterminants because, by construction, we have just acted with an element of the conformal group, and therefore we still have operators in the same supermultiplet. This can be explicitly seen by acting on the corresponding supersymmetry constraint equation. We know the action of half of the Q on the subdeterminant should vanish, hence

$$[Q, \det{}_n(\Phi)] = 0 \longrightarrow U\,[Q, \det{}_n(\Phi)]\,U^{-1} = 0\,, \tag{4.16}$$

that in turn implies that

$$\left[Q', \det{}_n(\Phi)'\right] = 0\,, \tag{4.17}$$

where we have used the general form of the transformation law $Q' = UQU^{-1}$, where U is the relevant representation of the group element Λ. To be more precise, in the particular case of the infinitesimal transformation (4.11) acting for example on Q^I_α (where I is in the SU(4) index and α is a Weyl-spinor index), we get

$$Q'^I_\alpha = Q^I_\alpha + \beta_i \sigma^i_{\alpha\dot\alpha} \bar S^{I\,\dot\alpha}\,, \tag{4.18}$$

where β_i is the generator of the special conformal transformation (4.13). Observe that we have just mixed Q^I_α and $\bar S^{I\dot\alpha}$ that indeed form by themselves a representation of the conformal group (see for example [21]). Therefore, we can safely conclude that generalized giant gravitons correspond to half BPS states in the CFT.

Next, we move on into the question of the dual operators to multiparticle giant graviton states breaking one quarter of the supersymmetries proposed in section 3.2. As dual CFT operators to these composite giant graviton states, we propose the naive product of two subdeterminants, where the second one has been transformed by the map induced by the AdS_5 isometry, i.e.

$$\mathcal{O}_{(n,k)} = \det{}_n(\Phi) \times \det{}_k(\Phi)' , \tag{4.19}$$

where n and k are the R-charges of the first and second giant gravitons respectively. Note that this candidate operator has the correct R-charge $n+k$, and encodes the characteristic bound for each giant graviton. The R-symmetry properties of these composite states are those of the tensor product of the representations $(0,n,0)$ and $(0,k,0)$ in which lie the two giant gravitons. As shown in appendix A, the representations appearing in the above product are of the form (p,q,p), some of which are short and define one quarter BPS states [22], while the others are long representations not enjoying any protection from quantum corrections. The fact that the supergravity binary system is supersymmetric implies, from the AdS/CFT correspondence, that also the dual CFT operator should be supersymmetric. Hence, long representations should not occur in the above decomposition, and the resulting dual operator should be in a *short* (p,q,p) representation, preserving therefore exactly one quarter of the supersymmetry. This fraction coincides with the one found in the supergravity description of section 3.2. Although the above argument is not a complete proof, it is certainly a check that the proposed dual operator have passed.

Note that group theory arguments imply the supersymmetry of these operators only for the free theory. In the interacting case, quarter BPS operators are known to mix with other descendent non-BPS operators. Nevertheless, in our case the duality with the supergravity picture suggests that the operator $\mathcal{O}_{(n,k)}$ is one quarter BPS for the full interactive theory in the infinite N and infinite 't Hooft parameter limit.

As a last comment, note that our quarter BPS operators are made out of subdeterminants and therefore are valid for large R-charge, while those explicitly considered in the literature have small R-charge [22, 24].

5. Summary and discussion

In this article, we have found giant graviton configurations with generic motion in AdS_5. The D3-brane dynamics on the giant graviton embedding ansatz reduces to that of a massive point particle in AdS_5 and therefore all the corresponding solutions follow timelike geodesics. Due to the fact that all such geodesics are related via isomorphism transformations of the background fields, the most general giant graviton configuration can be found by acting on the original giant graviton solution of ref. [4]. In particular, to illustrate better this solution-generating mechanism, we considered the explicit example of a giant graviton orbiting on a great circle of the $\mathcal{S}^3$ inside AdS_5 at a constant radius. Nevertheless, we emphasize the fact that any timelike geodesic solves the problem and therefore more involved configurations, having multiple angular momenta and oscillating radial position, are possible.

Next, we proved that all these new solutions are one half BPS states, by a detailed analysis on the meaning of an isometry transformation for the κ-symmetry constraint of the D3-brane supersymmetric world volume theory. Again, for the case of the orbiting giant graviton, the explicit form of the κ-symmetry projector and of the surviving Killing spinor is given.

An interesting outcome of the above analysis is the observation that different giant gravitons, following different timelike geodesics, have different κ-symmetry projectors. It is therefore possible to break larger fractions of supersymmetry by considering multiparticle giant graviton states. To illustrate this mechanism, we explicitly constructed a quarter BPS binary system, with one giant graviton orbiting around the other. We would like to stress that this configuration implements a new kind of time-dependent supersymmetric solution in string theory, since time-dependent states will arise in the open string sector for strings stretched between the two giant gravitons (see [25] for a similar construction).

Also, using the AdS/CFT conjecture, we obtained the explicit form of the dual operator to generalized giant gravitons, and showed it is half BPS. In particular, this operator can be seen as the result of a conformal symmetry transformation on subdeterminant operators. Finally, we proposed the product of such operators to describe multiparticle giant graviton states in the CFT, and gave arguments in favor of this hypothesis.

It would be interesting to extend the present work to verify, by a direct calculation, that these multiparticle operators indeed preserve one quarter of the supersymmetries. We believe that, using these techniques, it should be possible to obtain other fractions of supersymmetry, like for example one eighth. As a matter of fact, one eighth BPS states can be singled out from products of three or more single trace half BPS operators.[7] Since in the low momentum limit the CFT dual of giant gravitons should reduce to single trace operators, it is tempting to conjecture that configurations of three giant gravitons preserving one eighth of the total supersymmetry could be found. The corresponding supergravity problem is to find a three giant graviton configuration preserving one eighth of the supersymmetries.

Finally, it is well-known that standard giant graviton condensates give rise to superstars [9]. In this article we did not consider the backreaction on the geometry due to the presence of the D-branes. Nevertheless, it an important subject; since generalized giant gravitons carry angular momentum in AdS_5, some of their condensates may form a rotating generalization of the superstar. Work is in progress to identify such a supergravity solution.

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A. Some properties of product states

Theorem 2. *In the decomposition of the tensor product* $(0,p,0)\otimes(0,q,0)$ *into irreducible representations, only representations of the form* (m,n,m) *occur.*

[7] See section 3.5 of [3] for a nice review, and references therein.

Proof. A $su(4)$ highest weight with Dynkin labels $(0, p, 0)$ can equally well be specified in terms of its partition $\{p; p\}$ corresponding to a Young tableau with two rows and p boxes in each row. To compute the tensor product $(0, p, 0) \otimes (0, q, 0)$ we can then apply the Littlewood-Richardson rule to the product of Young tableaux

$$\underbrace{\begin{array}{|c|c|c|c|c|}\hline \ & \ & \ & \cdots & \ \\ \hline \ & \ & \ & \cdots & \ \\ \hline \end{array}}_{p} \otimes \underbrace{\begin{array}{|c|c|c|c|c|}\hline \mathrm{a} & \mathrm{a} & \mathrm{a} & \cdots & \mathrm{a} \\ \hline \mathrm{b} & \mathrm{b} & \mathrm{b} & \cdots & \mathrm{b} \\ \hline \end{array}}_{q}$$

We first have to add the q boxes **a** of the right tableau to the left tableau; we add i boxes to the first row, and $q-i$ to the third row. No box can be added to the second row because we cannot have boxes with the same label in the same column. We obtain a Young tableau with partition $\{p+i; p; q-i\}$. Regularity imposes then $p \geq q - i$. We have now to add q boxes **b** to the resulting tableau. In counting from right to left and top to bottom, the number of boxes **a** must always be greater or equal to the number of boxes **b**. Therefore, no **b** box can be added to the first line; let us call j and k the number of **b** boxes added to the second and third line respectively. The remaining $q - j - k$ **b** boxes are inserted in the fourth line. We end with a Young tableau with partition $\{p+i; p+j; q-i+k; q-j-k\}$. Now, to keep the counting of **a** greater than the number of **b**, we must impose $j \leq i$ and $j + k \leq i$; the request that no two **b** boxes fall into the same column requires on the other hand $q - i + k \leq p$ and $q - j - k \leq q - i$. Finally, regularity of the tableau imposes $p + i \geq p + j \geq q - i + k \geq q - j - k$. Now, combining these inequalities, it follows that $i = j + k$, and the partition reads $\{p+j+k; p+j; q-j; q-j-k\}$. The first $q - j - k$ columns of the tableau have four rows, and can be ignored by eliminating $q - j - k$ boxes to each row (these representations are the long ones). Hence, the most general Young tableau appearing in the tensor product has partition $\{p - q + 2j + 2k; p - q + 2j + k; k\}$, or equivalently, Dynkin labels $(k, p - q + 2j, k)$, of the form (m, n, m) as stated. □

Note that short $(0, q, 0)$ states have scaling dimension $\Delta = q$, and short (p, q, p) states have scaling dimension $\Delta = 2p + q$. The short representation $(k, p - q + 2j, k)$ has conformal scaling dimension $\Delta = p - q + 2j + 2k$, and requiring it to be equal to the conformal dimension $p + q$ of the product $(0, p, 0) \otimes (0, q, 0)$, we obtain the relation $2j + 2k = p + 2q$. In other words, the only short SU(4) multiplets appearing in the tensor product are of the form $(k, 2p + q - 2k, k)$ with k positive integer subject to the constraints $k \leq q$ and $2k \leq 3p$.

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